Ministry of Higher Education and Scientific Research

University of Science and Technology of Oran Mohammed Boudiaf (USTO-MB)

Chapter 2: Statics of Fluids (Pressure force & Archimedes thrust)

Course : Licence
Speciality: Electromechanical
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1. INTRODUCTION

- During a scuba dive, we find that the water pressure increases with depth.
- The water pressure at the bottom of a dam is greater than near the surface. The effects of pressure must be taken into consideration when dimensioning structures such as dams, submarines, reservoirs …

2. Assumptions

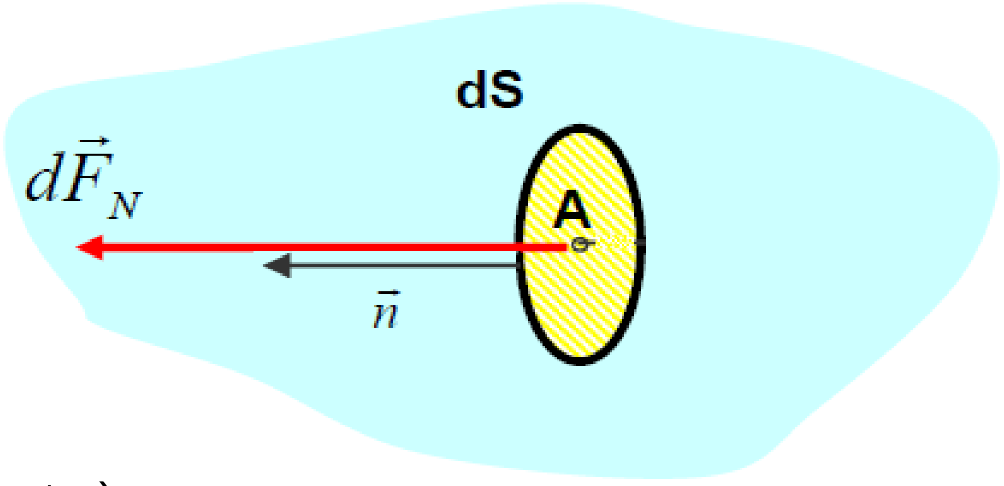
- In this chapter, we consider that:
- The studied fluids are at rest and in equilibrium.
- No friction between the molecules of the fluid (the fluid is assumed to be perfect).

3. CONCEPT OF PRESSURE AT A POINT OF A FLUID

- The pressure represents the intensity of the normal component of the force exerted by the fluid on the unit area.

$$\overrightarrow{dF} = p \cdot dS \cdot \vec{n}$$

$$P_A = \frac{\left\|\overrightarrow{dF_N}\right\|}{dS}$$



dS : Elementary surface of the facet with center A (in square meter),
n: Unit vector of the normal in A,
dF **:** Normal component of the elementary pressure force.
P A : pressure at point A (in Pascal),

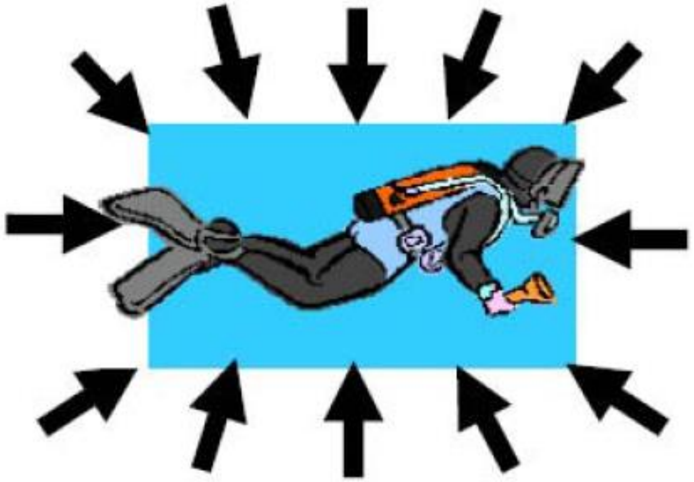
Explanation : Each cm² surface of our skin supports about 1 kg (force) representing the weight of the atmosphere. It is the atmospheric pressure at sea level.

Example: Each cm² of surface of our skin supports approximately 1 kg (force) representing the weight of the atmosphere. It is the atmospheric pressure at sea level. We do not feel it because our body is incompressible and its cavities (stomach, lungs, etc.) contain air at the same pressure.

If we rise 5,000 m, the atmospheric pressure is twice as low as at sea level because the mass of air above our head is then half as low. Hence the need for aircraft pressurization.

In scuba diving, to measure the pressure, we

1 bar = 1 kg/cm².



3.1. Pressure units

- Pascal (N/m^2), MPa (N/mm^2), Bar, atmosphere .
- 1 [bar] = 10^5 [Pa]
- 1 [atm] = 1.01325 10^5 [Pa]
- 1 [atm] = 1.01325 [bar]
- 1 [bar] = 1 kgf /cm^2, (1 kgf = 9.81N)

4. Fundamental relationship of hydrostatics

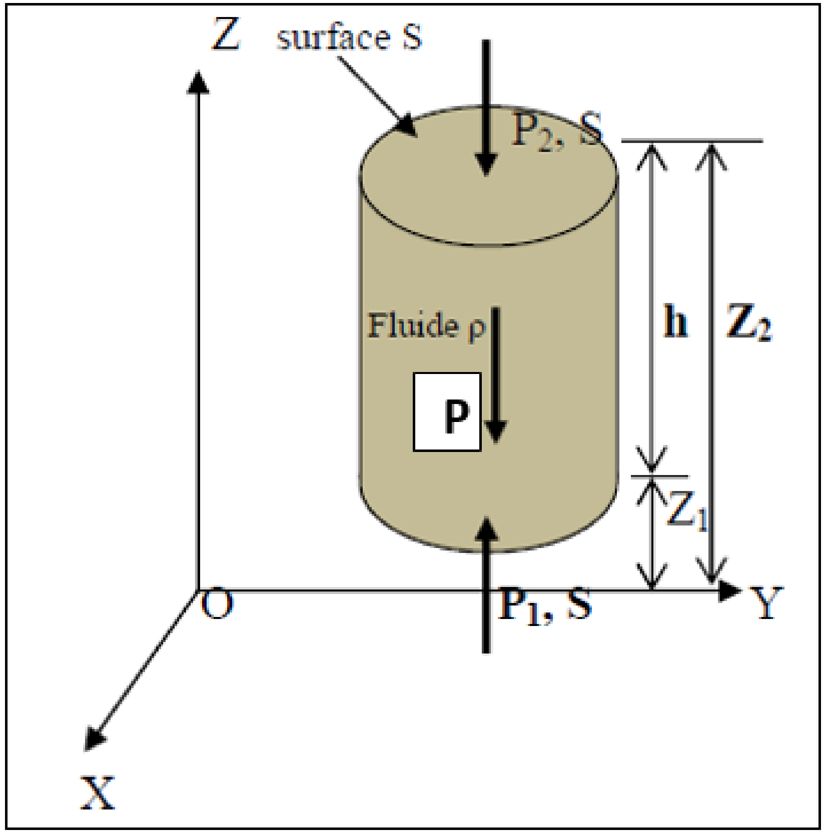
- We consider a fluid element of density **ρ** representing a cylindrical vertical column of constant cross-section S .

✓ Force due to P_1 : $F_1 = P_1 .S$

✓ Force due to P_2 : $F_2 = P_2 .S$

✓ Force due to the weight of the liquid column:

$$W = mg = \rho g V = \rho g S\,(Z_2 - Z_1)$$



- The equilibrium condition is then written :

$$F_1 - F_2 - W = 0 \Rightarrow P_1 S - P_2 S - \rho g S(Z_2 - Z_1) = 0$$

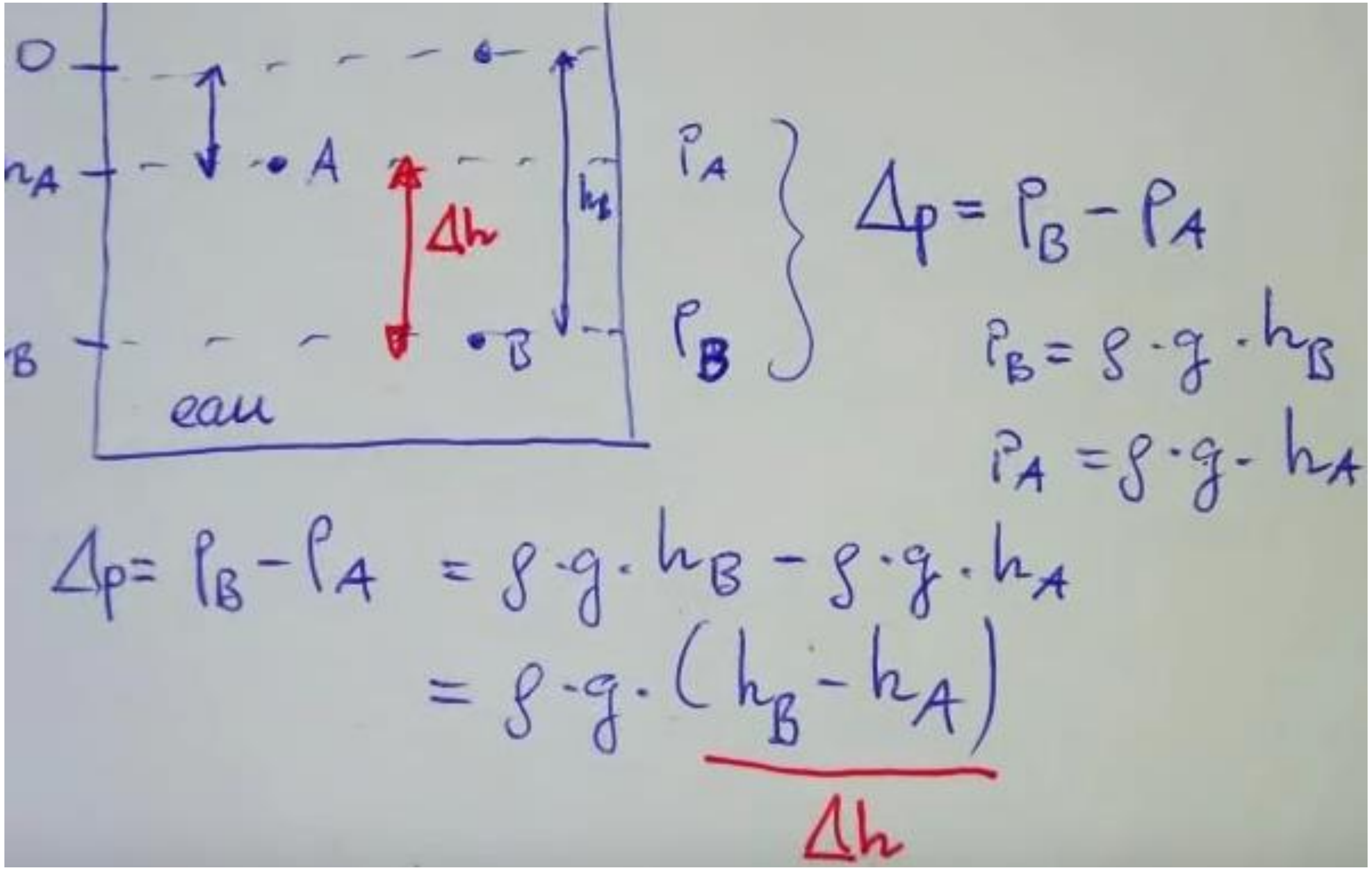
- SO :

$$\boxed{\mathbf{P_1 - P_2 = \rho g(Z_2 - Z_1)}}$$

And therefore the law of the statics of fluids is given by:

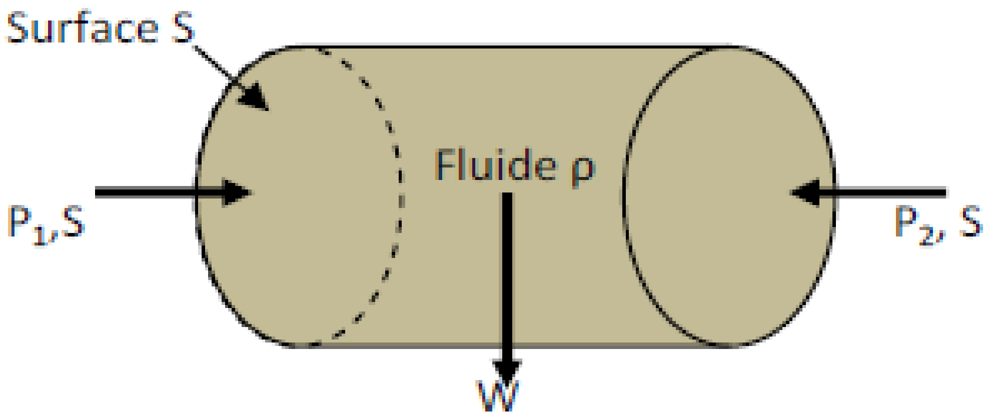
$$P_1 - P_2 = \rho g(Z_2 - Z_1) \Rightarrow P_1 + \rho g Z_1 = P_2 + \rho g Z_2 \Rightarrow \frac{P_1}{\rho g} + Z_1 = \frac{P_2}{\rho g} + Z_2$$

4.1. Difference in pressure between two points of a liquid



4.2. The pressures on the horizontal plane:

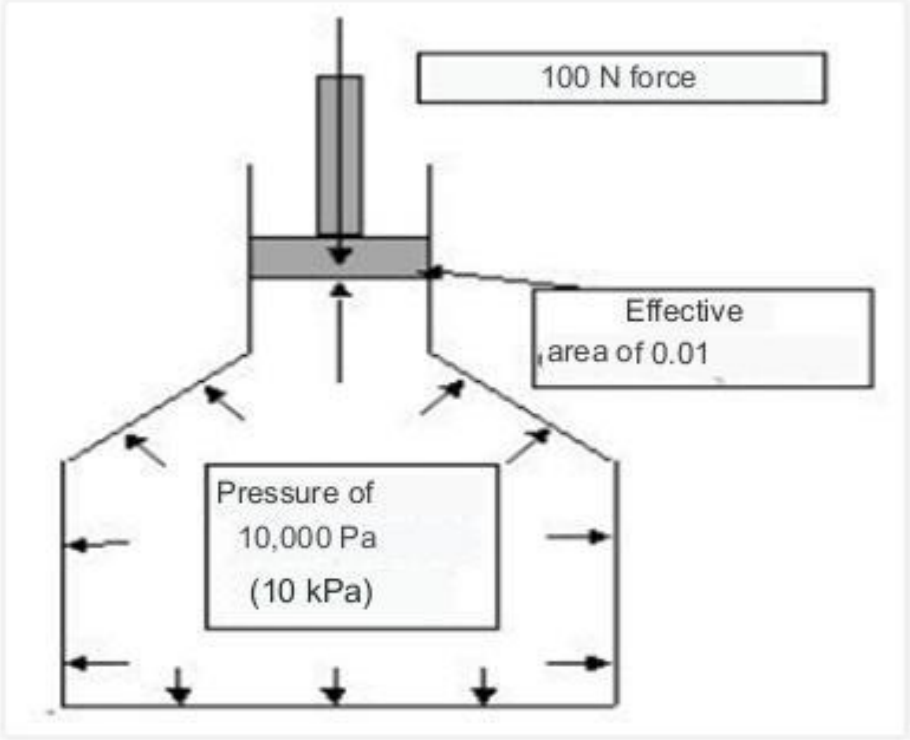
$$P_1S - P_2S + 0 = 0 \Rightarrow P_1 = P_2$$



We conclude that: On the same horizontal plane , all pressures are equal (therefore Isobaric Pressures).

5 . PASCAL 'S THEOREM :

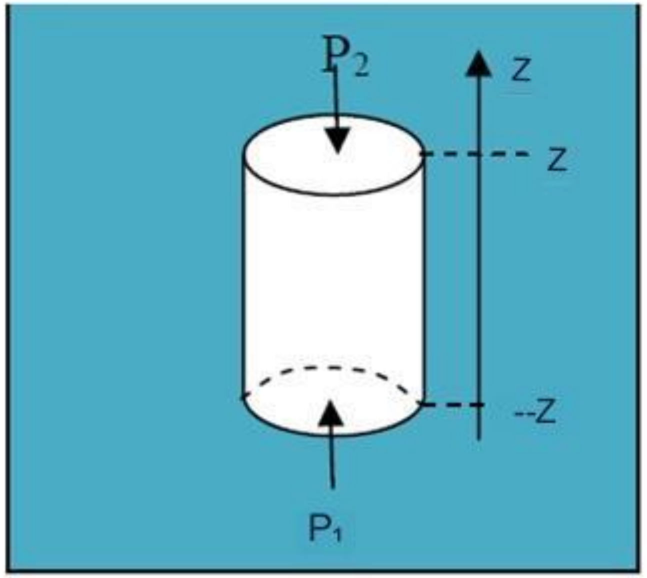
- In the 17th century, Blaise Pascal enunciated a law:
- "At rest, the pressure of a fluid at a point is
- the same in all directions.



6. Archimedes push

Consider a closed surface forming a solid body of density p, and volume

V immersed in a fluid of density p



Solid body immersed in a fluid

The vertical forces acting on the volume element are due to hydrostatic pressures. The result of these forces is:

$$F_R = (P_1 - P_2)\,S = \rho g(Z_2 - Z_1)S = \rho g W$$

Therefore, a body immersed in a fluid is subjected to the action of vertical thrust opposite in direction and equal to the weight of the fluid displaced by the body:

$$F_A = \rho g W$$

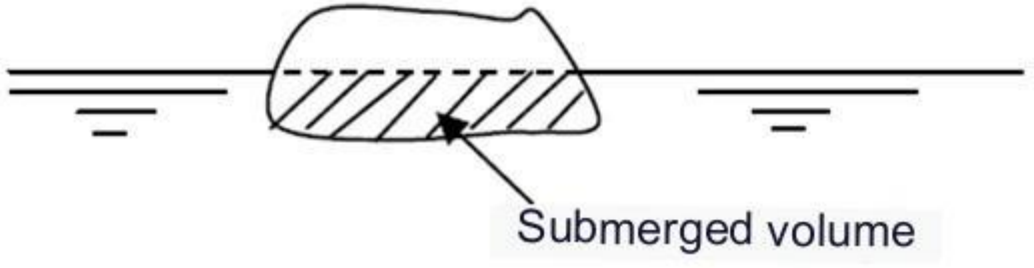
The force F is called the Archimedes force.

Archimedes' theorem :

Any body immersed in a fluid receives from this fluid a vertical force (thrust) , upwards, the intensity of which is equal to the weight of the volume of fluid displaced (this volume is therefore equal to the submerged volume of the body). It follows from this theorem that if the weight of a body placed in a fluid mass is
less than the weight of its fluid volume, the body floats .

Example :

What is the volume fraction of a piece of solid metal with specific gravity 7.25 floating on the surface of a container of mercury with specific gravity 13.6.



Solution :

The piece of metal in equilibrium when the weight of this piece equals the force of Archimedes:

$$F_A = \text{Weight} \Rightarrow \rho_{Hg} g\, V_{immer} \quad = \rho_{métal} g\, V_{total}$$

SO :

$$\frac{V_{immer}}{V_{total}} = \frac{\rho_{mètal}}{\rho_{Hg}} = \frac{7{,}25}{13{,}6} = 0{,}533$$

The volume fraction immersed in mercury is 53.3%

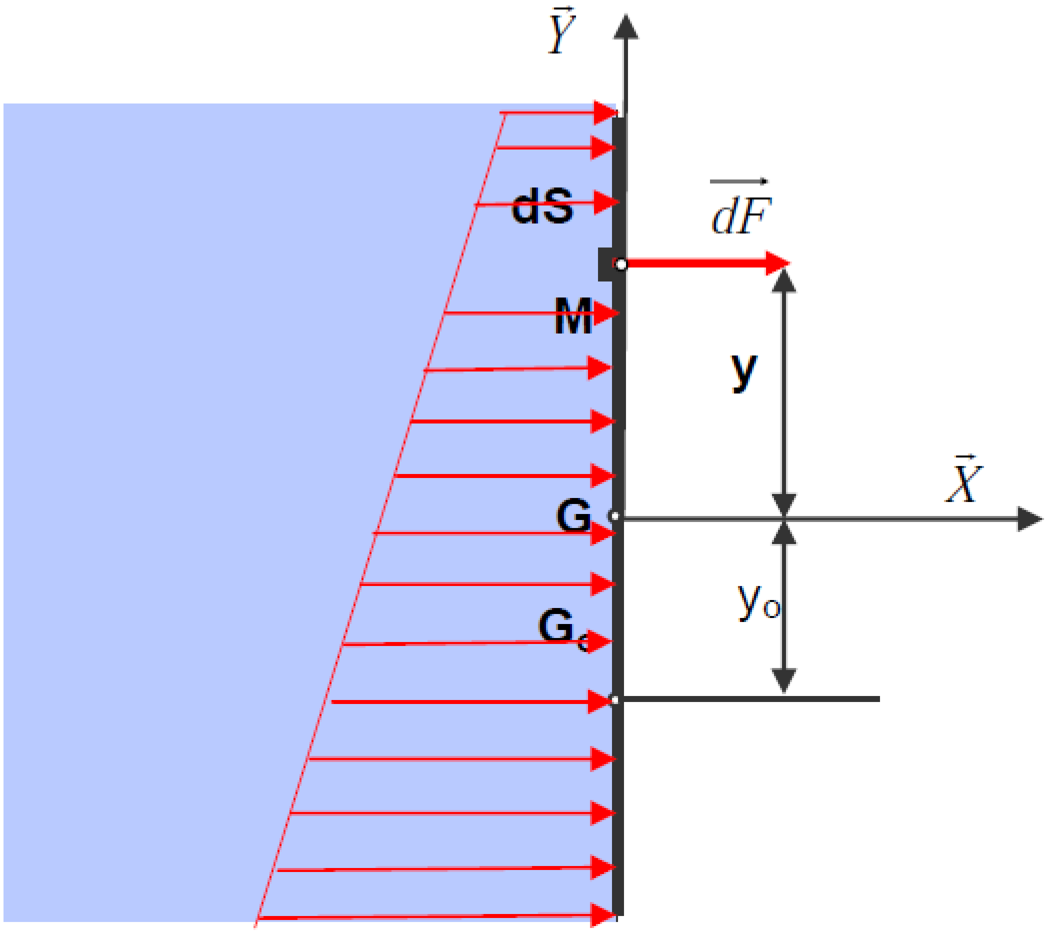
7. Force of pressure on the walls:

This force is defined as being the pressure force exerted by a fluid at rest on a contact surface, this force is always normal to the surface. The calculation of the hydrostatic forces on any surface immersed in water consists in determining the intensity of the force and its point of application.

Hypotheses

The vertical wall has an axis of symmetry (G, Y). G is its center of area .

On one side of the wall there is a fluid of density ϖ , on the other side there is air at atmospheric pressure P_{atm} . We denote by P_G the pressure at the center of surface G on the fluid side.



Consider a surface element of the plate ≪ dS ≫, the pressure exerted on this element is :

$$P = \rho \, g \, h$$

the final expression of F becomes:

$$F = \rho g \, h_G S$$

ho is the depth of the center of gravity of the surface

S is the surface area

The point of application of the resultant force F is called : the center of thrust.

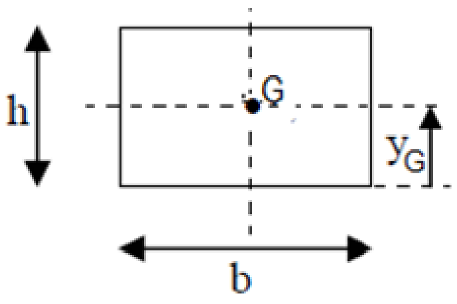
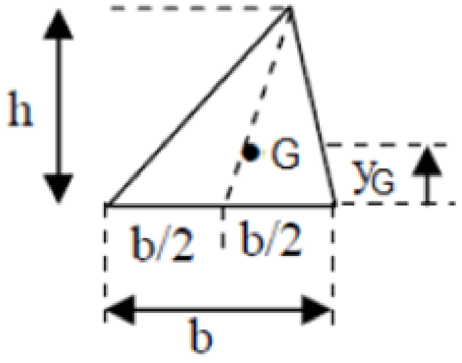
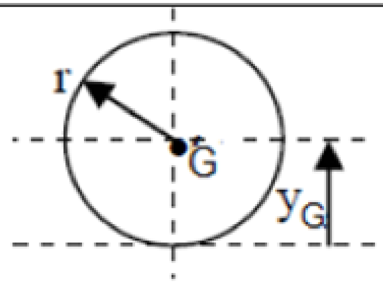
The formula for Yp becomes :

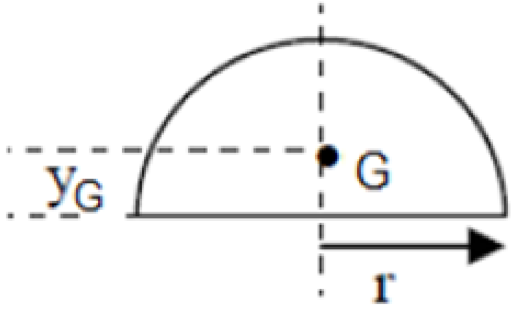
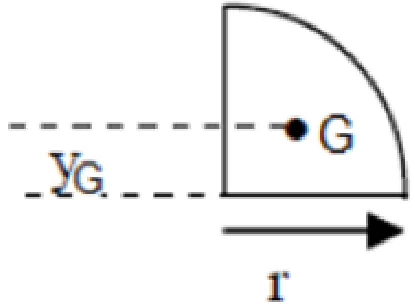
$$y_P = y_G + \frac{I_0}{y_G S}$$

This formula shows that the point of application of the resultant F is always more lower than the center of gravity by a distance equal to:

$$\frac{I_0}{y_G S}$$

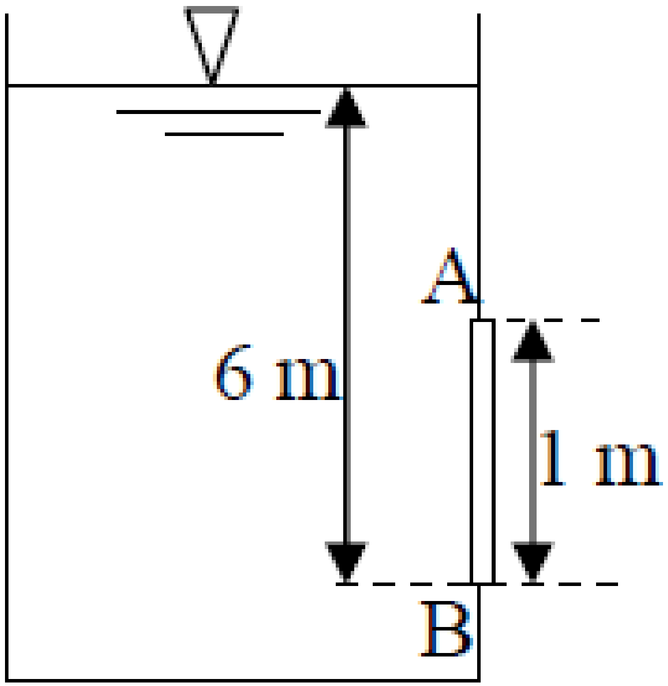
The following table provides this flat surface :

Type of surface	Geometric shape	Center of gravity	surface	Moment of inertia I_{xG}
Rectangle		$\frac{h}{2}$	bh	$\frac{bh^3}{12}$
Triangle		$\frac{h}{3}$	$\frac{bh}{2}$	$\frac{bh^3}{36}$
Circle		r	πr^2	$\frac{\pi r^4}{4}$

Half circle		$\frac{4r}{3\pi}$	$\frac{\pi r^2}{2}$	$\left(\frac{\pi}{8}-\frac{8}{9\pi}\right)r^4$
Quarter of circle		$\frac{4r}{3\pi}$	$\frac{\pi r^2}{4}$	$\left(\frac{\pi}{16}-\frac{4}{9\pi}\right)r^4$

Example :

Determine the hydrostatic thrust on the circular wall AB and its center of thrust. We give p=1000kg/m³ and g=9.81 m/s²



Solution :

1. Hydrostatic force

$$F = \rho g \, h_G S$$

$$F = 1000 \times 9.81 \times (6 - 0{,}5)\pi \, 0{,}5^2$$

$$\boldsymbol{F = 42,354 \; kN}$$

2. The center of thrust

$$h_P = y_P = y_G + \frac{I_{xG}}{y_G S}$$

$$y_G = h_G = 5{,}5 \; m \; et \; I_{xG} = \frac{\pi D^2}{64} = 0{,}049 \; m^4$$

$$h_P = 5{,}511 \; m$$