
Exercise series N°01

Exercise 1:

Let P, Q and R three propositions

1. Write the truth table of the following formula $P \wedge \overline{Q} \Rightarrow P \wedge Q$.
2. Using the truth table, prove that: $P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$; $P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$; $(P \Rightarrow Q) \equiv \overline{P} \vee Q$.
3. Without using the truth table, prove that:

$$\left((\overline{P} \vee Q) \wedge (P \vee \overline{Q})\right) \equiv (P \Leftrightarrow Q)\,.$$

Exercise 2: Write using the quantifiers $\forall$, $\exists$ the following assertions:

1. $A = \emptyset$, $A \cap B \neq \emptyset$, $A \subset B$, where A, B two sets in $\mathbb{R}$.
2. For all real number x, there exists a real number y, such that the square of x is greater than y.

Exercise 3:

1. Give the negation of the following mathematical statement:
   $\forall \varepsilon > 0$, $\exists N \in \mathbb{N}$, $\forall n \in \mathbb{N}$, $\forall p \in \mathbb{N}$, $n > N$ and $p > 0 \Rightarrow |U_{n+p} - U_n| < \varepsilon$
2. Let $f : \mathbb{R} \to \mathbb{R}$ is a function, give the contrapositive of the following assertions
   - $\forall x \in \mathbb{R},\ f(x) > 0 \Rightarrow x > 0$.
   - $\forall \epsilon > 0,\ \exists \eta > 0,\ \forall (x, y) \in \mathbb{R}^2,\ (|x - y| \leq \eta \Rightarrow |f(x) - f(y)| \leq \epsilon)$.

Exercise 4: Among the following propositions, determine which ones are true or false while justifying your answer.

- $\exists x \in \mathbb{N},\ 2 < x < 4$
- $\forall a, b \in \mathbb{R}_+$, if: $\dfrac{a}{a+1} = \dfrac{b}{b+1} \Rightarrow a = b$
- Let n be a positive integer, if n^2 is odd, then n is odd.
- $\forall n \in \mathbb{N}$, $\exists p \in \mathbb{N}$ such that $n(n+1) = 2p$

Exercise 4:

1. Using the Contrapositive reasoning, show that $\forall n \in \mathbb{N}$, n is a prime number $\Rightarrow n = 2$ or n is odd.
2. Using the Contradiction reasoning, prove that $\forall n \in \mathbb{Z}$ and all primes p, if $p \mid n$ then $p \nmid n + 1$.
3. Using a counter example, show that the following propositions are false
   - $\forall a, b \in \mathbb{R},\ (a + b)^2 = a^2 + b^2$.
   - $\forall n \in \mathbb{N},\ 2n - 3 \in \mathbb{N}$.
4. Using disjunction reasoning(case by case), prove that

$$\forall x \in \mathbb{R},\ |x - 1| \leq x^2 - x + 1.$$

5. Prove by recurrence that

$$\forall n \in \mathbb{N},\ (1 + a)^n \geq 1 + na.$$