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Chapter 2 Dependability Attributes: RMA (Part I)

Introduction

After having seen in chapter 1 the general elements of the dependability which are : threats, Means and Attributes, this chapter looks in detail at some quantitative attribute estimation techniques, in particular the RMA attributes (reliability, maintainability and availability).

1. Reliability R

1.1. definition

Reliability is the probability that an entity can perform a required function, under given conditions, during a given time interval $[t_1 , t_2]$; written as $R[t_1 , t_2]$. We are interested in a duration, not an instant. By assumption, the system works at the initial instant, theproblem is to know for how long. In general $[t_1 = 0]$ and $R(t)$ is the reliability.

1.2. Failure function

$$F(t)=1-R(t)$$

1.3. Mean time between failures (MTBF)

MTBF (Mean Time Between Failure) is often translated as mean time between failure. In other words, it corresponds to the expectation of the lifetime t.

$$MTBF = \int_0^\infty R(t)$$

Physically, MTBF can be expressed as the ratio of the times

MTBF= Σ uptime between n failures / number of n necessary maintenances or failures

So if after n failures we designate:

τi :Equipment uptime.

τir : Equipment repair time.

The M.T.B.F. does not consider downtime and repair time and is given by :

$$MTBF = T = \frac{1}{n}\sum_{1}^{n} \tau i$$

Example: A router has operated for 8,000 hours in continuous service, with 5 failures lying in 7, 22, 8,5, 3,5and 9 hours respectively. Determine its MTBF.

$$MTBF = \frac{8000 - (7 + 22 + 8,5 + 3,5 + 9)}{5} = 1590\ heures$$

1.4. Instantaneous failure rate λ

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This is the probability ($0 \leq R \leq 1$); a product must satisfactorily perform a required function, under given conditions and for a given period of time.
The mathematical expression of the failure rate at time t, $\lambda(t)$, is as follows
If this rate is constant λ instead of $\lambda(t)$, it can be estimated, instead of using probability ,by:
1/Mean Time Between Failures MTBF.

$$\lambda = 1/MTBF$$

Example: Determine λ of the previous router if it is assumed to be constant
$\lambda = \frac{1}{MTBF} = 6.289.10^{-4}\ défaillances/heures$

8 components operate in the same conditions during 550 h. The first component fails irreparably after 65 h of operation, the second after 115 h, the third after 135 h, component four after 340 h, component 5 after 535 h, the other three components continue to operate normally.
Lambda=
MBTF= (65+115+135+340+535+550*3)/8
Failure rate lambda=1/MTBF=

1.5. Reliability assessment R

1.5.1. through statistics

To evaluate systems of a certain brand (e.g. laptops), we study a large sample size.

R(t)=nbr of sample systems that have not failed since t=t-dt=0/ nbr total systems(sample size)

1.5.2 by a probabilistic law

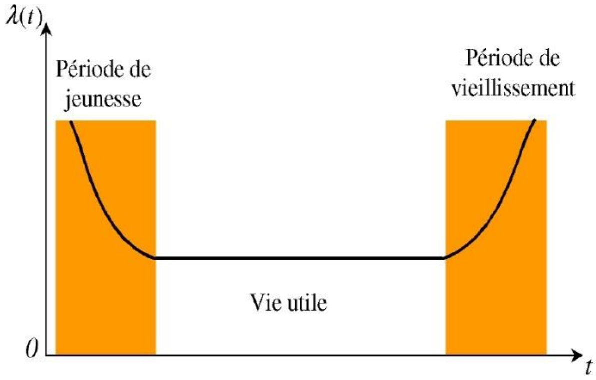
There are several laws for assessing reliability as a function of rates, such as the exponential law, fish law, etc.

The exponential law of R is :

$$R(t) = e^{-\lambda . t}$$

1.6. System life cycle

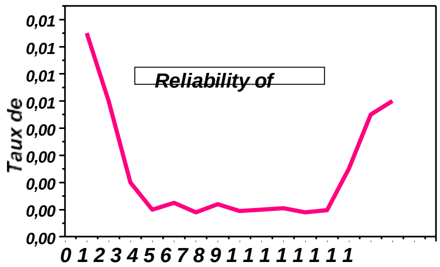
The evolution of a product's failure rate over its lifetime is characterized by what is known in reliability analysis as the bathtub curve.
The failure rate is high at the beginning of the device's life.
Then, it diminishes fairly rapidly over time (decreasing failure rate), this phase of life is called the youth period.
After that, it stabilizes at a value that we want to be as low as possible for a period called the useful life period (constant failure rate). At the end, it rises again as wear and ageing take their toll: this is the Ageing period (increasing failure rate).



Example:

A network is studied after 16500 hours. During this period, the network has accumulated 218 stops. The data are summarized in the table below. We want to know how the network's reliability and wear phase evolve as a function of downtime intervals.

hours	MTBF	Rate of failure
1000	66.7	0.015
2000	100	0.01
3000	250	0.004
4000	500	0.002
5000	400	0.0025
6000	555.6	0.0018
7000	416.7	0.0024
8000	526.32	0.0019
9000	500	0.002
10000	476.2	0.0021
11000	555.6	0.0018
12000	512	0,001953125
13000	200	0.005
14000	111.1	0.009
15000	100	0.01



Solution:

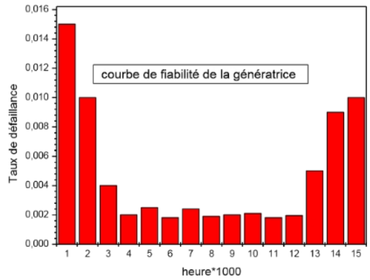


Figure: Generator reliability and bathtub curve

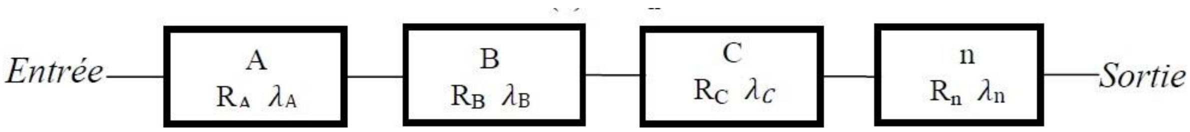
We can see that the network starts to deteriorate from 12000=12*1000 hours of operation. The bathtub behavior of the failure rate indicates more or less normal operation.

1.7. Reliability of multi-component systems

1.7.1. In series

The reliability Rs of a set of n components connected in series is equal to the product of the respective reliability RA, RB, RC, Rn of each component.

Rs = RA * RB * RC *...*Rn



In more detail, the overall lambda can be calculated as a function of the system's MTBF(S).

$$R(s) = (e^{-\lambda_A t}) * (e^{-\lambda_B t}) * (e^{-\lambda_C t}) * \ldots (e^{-\lambda_n t})$$

$$MTBF(s) = \frac{1}{\lambda_A + \lambda_B + \lambda_C + \ldots + \lambda_n}$$

Example 1:

Given a machine consisting of four components connected in series, a power supply RA=0.95, a receiver RB=0.92, an amplifier RC=0.97 and a loudspeaker RD= 0.89, determine the RS reliability of the device.

RS= RA. RB .RC. RD=0.95x 0.92x0.97x0.89=0.7545 (approx. 75% reliability)

Example 2:

Let's take a printer made up of 2000 components connected in series, all assumed to have the same reliability, very high R= 0.9999,

a) Determine device reliability.

$R(s)=R^n = 0.9999^{2000} = 0.8187$ (i.e. a reliability of about 82%)

b) If we halve the number of components R(s)=
$R^n = 0.9999^{1000} = 0.9048$ (approx. 90.5%)

c) If we want 90% reliability for all 2,000 components assembled in series, let's determine how reliable each component must be.

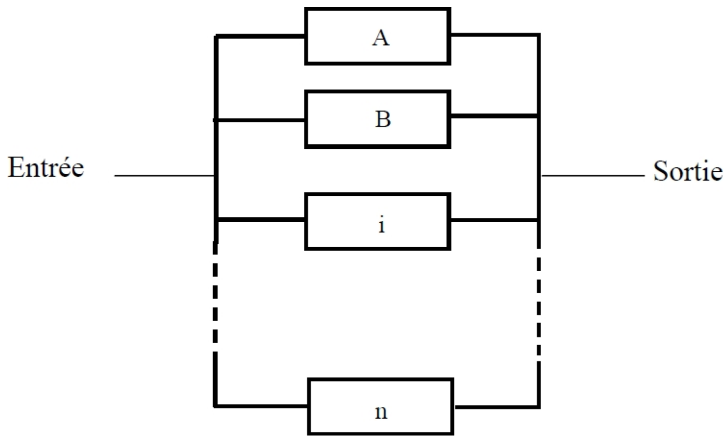
Equation: $R_S = 0.9000 = R^{2000}$

Expression can be written using logarithm so R=0.999945

$LnR_S = Ln0.9 = 2000.\ln R$

1.7.2. in parallel

The reliability of a system can be increased by placing components in parallel.



A device made up of **n** components in parallel can only fail if all **n** components fail at the same time. The reliability Rs of the system is calculated as a function of the unreliability of each component i, Fi=1-Ri by :

Example: Rs=1-[(1-R1)(1-R2)]

$R_s=1-\Pi(1-R_i)=1-((1-R_1)\ (1-R_1)\ (1-R_2)\ ..(1-R_i)...\ (1-R_n))$ i=1..n

Example:

Three routers A, B and C with the same reliability (homogeneous) RA= RB= RC=0.75 are connected in parallel.

a) Let's determine the reliability of the system

Rs=1-(1-0 ,75)^3=0.984

b) What number of parallel routers would be required to achieve an overall reliability of 0.999 (99.9%)?

Rs=0.999

Hence **1-(1-0 ,75)^n=0**.999

so 0.25^n=0.001

Using logarithms ln

ln(0.25)=ln(0.001)

N=4,9

This implies having at least 5 routers in parallel

c) If we wish to have an overall reliability of 99% with only three routers in parallel, what should be the reliability R of each of these routers?

0.99=1-**(1-Rx)^3**

Hence **(1-Rx)$^{3=0}$**.01

Applying the cube root of the two sides or the logarithm ln , we obtain at least Rx=0.7846=78.46%

1.1.1. In parallel, where m out of n components are required for a successful system

If we assume that the system consists of n components , all of equal reliability R, and that there mustbe at least m components in working order, the reliability of the assembly is given by :

$$R_S = \sum_{i=m}^{n} \left(\frac{n!}{i!(n-i)!}\right) R^i (1-R)^{n-i}$$

Example:

Case with 3 components with a minimum of 2 active components out of the 3 available at the start. System failure of only 1 component out of the three is tolerated. There must be at least two components in operation or active to accomplish the mission, the previousrelationship gives with n=3 and m=2

$3!/(2!)\ r^2 * (1-r)$

Example 2:

case with 4 components in parallel, with a minimum of 2 active components out of the four initially available. The system can tolerate the failure of two of the four components. There must be at least two components in operation or active to accomplish the mission, and the previous relationship gives with n=4 and m=2

$$R_S = R^4 + 4R^3(1-R) + 6R^2(1-R)^2 = 3R^4 - 8R^3 + 6R^2$$