University of science and technology Oran M-B

Academic year 2023/2024

Department of Mathematics

Algebra 1

# Exercise series $N^{\circ}05$

#### Exercise 1:

For the following pairs of polynomials f(X) and g(X), find the quotient and remainder on dividing g(X) by f(X).

1.  $g(X) = X^7 - X^3 + 5$ ,  $f(X) = X^3 + 7$  over  $\mathbb{Q}$ .

2. 
$$g(X) = X^2 + 1$$
,  $f(X) = X^2$  over  $\mathbb{Q}$ .

3.  $4X^3 - 17X^2 + X - 3$ , f(X) = 2X + 5 over  $\mathbb{R}$ .

## Exercise 2:

Determine the GCD of the following polynomials

 $X^{3} - X^{2} - X - 2$ , and  $X^{5} - 2X^{4} + X^{2} - X - 2$ .  $X^{4} + X^{3} - 2X + 1$  and  $X^{3} + X + 1$ .

#### Exercise 3:

For each of the following polynomials (over the given field) say, with justification, whether it is reducible or irreducible.

- 1.  $X^2 + X + 1$  in  $\mathbb{K}_5[X]$ .
- 2.  $X^2 + X + 1$  in  $\mathbb{K}_7[X]$ .

### Exercise 4:

Determine the multiplicity of the root 1 of the polynomial

$$P(X) = X^5 - 5X^4 + 14X^3 - 22X^2 + 17X - 5.$$

## Exercise 5:

For which values of  $\alpha \in \mathbb{R}$ , the polynomial  $P(X) = X^3 - 3X + \alpha$  has a root with multiplicity 2. What is the other root.