
Exercise series N°05

Exercise 1:

For the following pairs of polynomials $f(X)$ and $g(X)$, find the quotient and remainder on dividing $g(X)$ by $f(X)$.

1. $g(X) = X^7 - X^3 + 5$, $f(X) = X^3 + 7$ over \mathbb{Q} .

2. $g(X) = X^2 + 1$, $f(X) = X^2$ over \mathbb{Q} .

3. $4X^3 - 17X^2 + X - 3$, $f(X) = 2X + 5$ over \mathbb{R} .

Exercise 2:

Determine the GCD of the following polynomials

$$X^3 - X^2 - X - 2, \text{ and } X^5 - 2X^4 + X^2 - X - 2.$$

$$X^4 + X^3 - 2X + 1 \text{ and } X^3 + X + 1.$$

Exercise 3:

Determine the multiplicity of the root 1 of the polynomial

$$P(X) = X^5 - 5X^4 + 14X^3 - 22X^2 + 17X - 5.$$

Exercise 4:

For each of the following polynomials (over the given field) say, with justification, whether it is reducible or irreducible.

1. $X^2 + X + 1$ in $\mathbb{K}_5[X]$.

2. $X^2 + X + 1$ in $\mathbb{K}_7[X]$.

Exercise 5:

For which values of $\alpha \in \mathbb{R}$, the polynomial $P(X) = X^3 - 3X + \alpha$ has a root with multiplicity 2. What is the other root.