University of science and technology Oran M-B
Department of Mathematics
Faculty of mathematics and computer science
The 18^{th} January, 2024

Final exam of Algebra 1 (Duration 1h30)

Exercice 01:(07 points)

1. Give the truth table of the following proposition
$$(R \Rightarrow \overline{P}) \wedge \left((\overline{Q} \Rightarrow P) \vee R\right).$$
Where P, Q and R are propositions.

2. Is the following proposition true or false? Justify.
$$\forall x \in \mathbb{R} - 1,\ \exists n \in \mathbb{N},\ \frac{n}{x-1} + 1 > 0.$$

3. Let f be an application from $\mathbb{R}$ to $\mathbb{R}$ defined by $f(x) = x - x^2$. Is f injective? surjective? (Provide the definitions and the justification of your answer).

4. Let f be a group-homomorphism from $(G, *)$ to $(H, \star)$, show that f is injective if and only if $Ker(f) = \{e_G\}$, where e_G is the identity element of $(G, *)$.

5. Is $(\mathbb{Z} \setminus 6\mathbb{Z}, \dot{+}, \dot{\times})$ a field? Justify.

Exercice 02:(06 points) Let $\mathcal{R}$ be a relation defined on $\mathbb{R}$ by
$$x\mathcal{R}y \ \Leftrightarrow \ x - |x| = y - |y|.$$

1. Prove that $\mathcal{R}$ is an equivalence relation.

2. Determine the equivalence class of a real a.

3. Determine the quotient set $\mathbb{R} \backslash \mathcal{R}$.

Exercice 03:(07 points) Define on $\mathbb{R}$ the binary operation $*$ by
$$\forall x, y \in \mathbb{R},\ x * y = x + y - 2.$$

1. Show that $(\mathbb{R}, *)$ is an abelian group.

2. Let $H = \{x \in \mathbb{Z},\ x \textit{ is pair}\}$. Show that H is a subgroup of $(\mathbb{R}, *)$.

3. Let
$$\begin{aligned} f :&(\mathbb{R}, *) \to (\mathbb{R}, +) \\ &x \longmapsto kx - 2. \end{aligned}$$
Determine the values of $k \in \mathbb{R}$ for which f is a group homomorphism.

Good luck