University of science and technology Oran M-B
Department of Mathematics

Academic year 2023/2024
Algebra 2

Exercise series N°02

Exercise 01:

Are the following applications linear or not?

$$\begin{aligned} f_1 : \mathbb{R}^2 &\longrightarrow \mathbb{R}^3 \\ (x,y) &\longmapsto (x+y, x-y, 0) \end{aligned} \quad , \quad \begin{aligned} f_2 : \mathbb{R}^2 &\longrightarrow \mathbb{R} \\ (x,y) &\longmapsto x^2 - y, \end{aligned} \quad \begin{aligned} f_3 : \mathbb{R}^2 &\longrightarrow \mathbb{R}^2 \\ (x,y) &\longmapsto (|x|+|y|, 2). \end{aligned}$$

Exercise 02:

Let the application f defined by:

$$\begin{aligned} f : \mathbb{R}^3 &\longrightarrow \mathbb{R}^2 \\ (x,y,z) &\longmapsto (-2x+y+z, x-2y+z). \end{aligned}$$

1. Prove that f is a linear application.
2. Determine a basis for $kerf$, and deduce $dimImf$.
3. Determine a basis for Imf.

Exercise 03:

Let the application f defined by:

$$\begin{aligned} f : \mathbb{R}^3 &\longrightarrow \mathbb{R}^3 \\ (x,y,z) &\longmapsto (x-z, 2x+y-3z, -y+2z). \end{aligned}$$

Let$\{e_1, e_2, e_3\}$ be the canonical basis of $\mathbb{R}^3$.

1. Compute $f(e_1)$, $f(e_2)$ and $f(e_3)$.
2. Determine the coordinates of $f(e_1)$, $f(e_2)$ and $f(e_3)$ in the canonical basis.
3. Compute a basis for $kerf$ and a basis for Imf.

Exercise 04:

Consider the linear application f defined by:

$$\begin{aligned} f : \mathbb{R}^3 &\longrightarrow \mathbb{R}^4 \\ (x,y,z) &\longmapsto (x+z, -x+y, y+z, x+y+2z). \end{aligned}$$

1. Determine a basis for Imf, and $dimImf$, then deduce $Rg(f)$.
2. Determine $Kerf$ and indicate if it is one-one application.

Exercise 05:

Let f be a linear application from E to E, where E is a vector space of dimension n, with n is even.
Prove that the following two assertions are equivalents:

1. $f^2 = 0_E$ (0_E is the null application), and $n = 2dimImf$.
2. $Imf = kerf$.

Exercise 06:

Let f a linear application from E to F, prove that

1. Suppose that f is one-one, let $\{u_1, u_2, ..., u_n\}$ a linearly independent family of E. Prove that $\{f(u_1), f(u_2), ..., f(u_n)\}$ a linearly independent family of F.
2. Suppose that f is onto, let $\{u_1, u_2, ..., u_n\}$ a generating family of E. Prove that $\{f(u_1), f(u_2), ..., f(u_n)\}$ is a generating family of F.