University of science and technology Oran M-B
Department of Mathematics

Academic year 2023/2024
Algebra 2

Exercise series N°03 and 04

Exercise 01:

Consider the following matrices A, B and C, such that

$$A = \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix}, \quad B = \begin{pmatrix} 0 & 2 \\ 2 & 1 \end{pmatrix}, \quad C = \begin{pmatrix} 2 & 0 \\ 4 & -2 \end{pmatrix}.$$

Compute $A + B - C$, $-2A + B$, BC, AB, BA and ${}^tB^tA$.

Exercise 02:

1. Compute all the possible products between 2 matrices

$$A = \begin{pmatrix} 4 & 2 \\ 6 & 4 \end{pmatrix}, \quad B = \begin{pmatrix} 2 & -2 \\ 2 & 4 \end{pmatrix}, \quad C = \begin{pmatrix} 1 & 2 & 0 \\ 3 & 1 & 4 \end{pmatrix}, \quad D = \begin{pmatrix} -1 & -1 & 0 \\ 1 & 4 & -1 \\ 2 & 1 & 2 \end{pmatrix}, \quad M = \begin{pmatrix} 2 \\ 4 \end{pmatrix}.$$

2. Determine the transpose matrix of the matrices A, C and M.

3. Compute the product of the following two matrices $\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$, $\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$. What do you conclude?

Exercise 03:

Find the trace and determinant of the following matrices

$$A = \begin{pmatrix} 1 & 2 \\ 0 & 4 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 3 & -5 \\ 2 & -1 & 4 \\ 2 & 0 & 1 \end{pmatrix}, \quad C = \begin{pmatrix} 8 & 1 & 0 & 1 \\ -3 & 2 & 1 & 0 \\ 2 & -2 & 0 & 0 \\ -1 & 5 & 4 & 0 \end{pmatrix}.$$

Determine $rk(C)$.

Exercise 04:

Let

$$A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$

Find all the matrices $C \in M_2(\mathbb{R})$ that commute with A, which means $AC = CA$.

Exercise 05:

Determine if the following matrices are invertible, for the 2×2 invertible matrices, determine their inverses and check by computing $A_i A_i^{-1}, \ i = 1, ..., 5$.

$$A_1 = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}, \quad A_2 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \quad A_3 = \begin{pmatrix} 2 & -4 \\ 4 & -8 \end{pmatrix}, \quad A_4 = \begin{pmatrix} 1 & 0 & 0 \\ 3 & 2 & 0 \\ 1 & -1 & 4 \end{pmatrix}, \quad A_5 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

Exercise 06:

Prove that A is invertible and determine it's inverse in function of powers of A and I_n.

$$2A^2 + 6A - 4I_n = 0, \ A^6 + I_n = 0.$$

Exercise 07:

Consider the following matrix

$$A = \begin{pmatrix} 2 & 0 & 4 \\ 6 & 4 & -2 \\ 0 & 2 & 0 \end{pmatrix}.$$

1. Prove that A is invertible.

2. Determine A^{-1} using co-factor method.

3. Determine A^{-1} using Gauss Jordan method.

Exercise 08:

Let f be a linear application defined by

$$\begin{aligned} f : \mathbb{R}^3 &\longrightarrow \mathbb{R}^3 \\ (x, y, z) &\longmapsto (2x + y + z, x + 2y + z, x + y + 2z). \end{aligned}$$

1. Write the matrix associated to f in the canonical basis B_0 of $\mathbb{R}^3$.
2. Consider the list B_1 of $\mathbb{R}^3$, where

$$B_1 = \{u(1,-1,0), v(1,0,-1), w(1,1,1)\}.$$

   - Prove that B_1 is a basis for $\mathbb{R}^3$.
   - Determine the transition matrix P from B_0 to B_1, and compute P^{-1}.
   - Write the matrix associated to f in the basis B_1.

Exercise 09:

1. Consider the following matrix

$$A = \begin{pmatrix} 3 & -10 \\ -2 & 8 \end{pmatrix}.$$

   Determine A^{-1}.

2. Deduce the resolution of the following systems

$$\begin{cases} 3x - 10y = 4 \\ -2x + 8y = 7 \end{cases}, \qquad \begin{cases} 3x - 10y = 15 \\ -2x + 8y = -5 \end{cases}$$

3. Solve the following systems

$$\begin{cases} 2x + y = -1 \\ 6x - 10y = -17, \end{cases} \qquad \begin{cases} 5x - 2y + 9z = -8 \\ -2x + y - 4z = 4 \\ 6x - 20y - 16z = 0, \end{cases} \qquad \begin{cases} 2x - y - 3z = 0 \\ -x + 2z = 0 \\ 2x - 3y - z = 0, \end{cases} \qquad \begin{cases} 3x - 2y = 5 \\ -6x + 4y = 7. \end{cases}$$