Supplementary Exercises

Exercise 01: Let $f : \mathbb{R}^3 \to \mathbb{R}^3$ a linear application, where its associated matrix in the canonical basis is

$$A = \begin{pmatrix} 3 & -1 & 1 \\ 0 & 2 & 0 \\ 1 & -1 & 3 \end{pmatrix}.$$

1. Determine $f(x, y, z)$ for all $(x, y, z) \in \mathbb{R}^3$.
2. Let $u_1 = e_1 - e_3$, $u_2 = e_2 + e_3$ and $u_3 = e_1 + e_3$, where $B = \{e_1, e_2, e_3\}$ is the canonical basis of $\mathbb{R}^3$.
   (a) Prove that $B' = \{u_1, u_2, u_3\}$ is a basis of $\mathbb{R}^3$.
   (b) Compute $f(u_1)$, $f(u_2)$ and $f(u_3)$, and deduce the matrix A' associated to f in the basis B'.
   (c) Write the transition matrix P from B to B', and compute its inverse P^{-1}.
   (d) Write the general formula that relies A, A' and P and check the obtained results of this formula with the question (b).

Exercise 02: Let the matrix

$$A = \begin{pmatrix} 1 & -1 & 2 & 0 \\ 1 & 3 & 0 & -1 \\ 1 & -2 & 3 & 4 \end{pmatrix}.$$

1. Denote f the linear application represented by the matrix A in the canonical basis.
2. Determine the equations system of $Kerf$, and a basis for $Kerf$, then its dimension.
3. Determine a basis for Imf, then its rank.