University of science and technology Oran M-B — Faculty of mathematics and computer science
Department of Mathematics
The 10^{th} June, 2024

Makeup exam of Algebra 1 (Duration 1h30)

Exercice 01:(05 points)
Let P, Q and R be three propositions:

1. Write the truth table of each of the following propositions
   - $A \equiv P \vee (Q \wedge \overline{R})$.
   - $B \equiv (P \wedge \overline{Q}) \Rightarrow \overline{R}$.
2. Give the negation of the proposition A, and of B.
3. Using the contrapositive, show that

$$n^3 \text{ is a multiple of } 3 \Rightarrow n \text{ is a multiple of } 3.$$

Exercice 02:(07 points)
Let f be an application from $\mathbb{R}$ to $\mathbb{R}$ defined by $f(x) = \frac{x}{1+|x|}$.

1. Prove that $f(\mathbb{R}) \subset]-1, 1[$. Is f onto? Justify.
2. Prove that f is one to one.
3. Determine $f^{-1}(\{\frac{1}{2}\})$.
4. Prove that f is bijective from $\mathbb{R}$ to $]-1, 1[$, and determine its reciprocal application f^{-1}.

Exercice 03:(08 points) Define on $\mathbb{R}$ the binary operation $*$ by

$$\forall x, y \in \mathbb{R},\ x * y = x + y + \frac{1}{10}.$$

1. Show that $(\mathbb{R}, *)$ is an abelian group.
2. Let

$$\begin{aligned} f :& (\mathbb{R}, *) \to (\mathbb{R}, +) \\ & x \longmapsto 5x + \frac{1}{2}. \end{aligned}$$

   Show that f is a group homomorphism.
3. Let $H = \{\frac{2n-1}{10},\ n \in \mathbb{Z}\}$. Show that H is a subgroup of $(\mathbb{R}, *)$.

Good luck