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Pendulums

A man enters a tall tower, needing to know its height. He notes that a long pendulum extends from the ceiling almost to the floor and that its period is $12.0\ s$.
(a) How tall is the tower?
(b) **What If?** If this pendulum is taken to the Moon, where the free-fall acceleration is $1.67\ m/s^2$, what is its period there?

(a) $$T = 2\pi\sqrt{\frac{L}{g}}$$

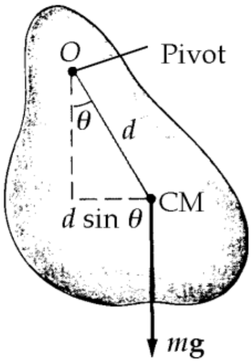
$$L = \frac{gT^2}{4\pi^2} = \frac{(9.80\ \text{m/s}^2)(12.0\ \text{s})^2}{4\pi^2} = \boxed{35.7\ \text{m}}$$

(b) $$T_{\text{moon}} = 2\pi\sqrt{\frac{L}{g_{\text{moon}}}} = 2\pi\sqrt{\frac{35.7\ \text{m}}{1.67\ \text{m/s}^2}} = \boxed{29.1\ \text{s}}$$

A physical pendulum in the form of a planar body moves in simple harmonic motion with a frequency of 0.450 Hz.

If the pendulum has a mass of 2.20 kg and the pivot is located 0.350 m from the center of mass, determine the moment of inertia of the pendulum about the pivot point.

$f = 0.450$ Hz, $d = 0.350$ m, and $m = 2.20$ kg



$$T = \frac{1}{f};$$

$$T = 2\pi\sqrt{\frac{I}{mgd}}; \quad T^2 = \frac{4\pi^2 I}{mgd}$$

$$I = T^2\frac{mgd}{4\pi^2} = \left(\frac{1}{f}\right)^2\frac{mgd}{4\pi^2} = \frac{2.20(9.80)(0.350)}{4\pi^2\left(0.450\ \text{s}^{-1}\right)^2} = \boxed{0.944\ \text{kg}\cdot\text{m}^2}$$

A "seconds pendulum" is one that moves through its equilibrium position once each second. (The period of the pendulum is precisely 2 s.) The length of a seconds pendulum is $0.9927\ m$ at Tokyo, Japan and $0.9942\ m$ at Cambridge, England. What is the ratio of the free-fall accelerations at these two locations?

The period in Tokyo is $T_T = 2\pi\sqrt{\dfrac{L_T}{g_T}}$

and the period in Cambridge is $T_C = 2\pi\sqrt{\dfrac{L_C}{g_C}}$

We know $T_T = T_C = 2.00\ \text{s}$

For which, we see $\dfrac{L_T}{g_T} = \dfrac{L_C}{g_C}$

or $\dfrac{g_C}{g_T} = \dfrac{L_C}{L_T} = \dfrac{0.994\,2}{0.992\,7} = \boxed{1.001\,5}$

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A rigid steel frame above a street intersection supports standard traffic lights, each of which is hinged to hang immediately below the frame. A gust of wind sets a light swinging in a vertical plane. Find the order of magnitude of its period. State the quantities you take as data and their values.

The swinging box is a physical pendulum with period $T = 2\pi\sqrt{\frac{I}{mgd}}$.

The moment of inertia is given approximately by

$$I = \frac{1}{3}mL^2 \text{ (treating the box as a rod suspended from one end).}$$

Then, with $L \approx 1.0$ m and $d \approx \frac{L}{2}$,

$$T \approx 2\pi\sqrt{\frac{\frac{1}{3}mL^2}{mg\left(\frac{L}{2}\right)}} = 2\pi\sqrt{\frac{2L}{3g}} = 2\pi\sqrt{\frac{2(1.0\text{ m})}{3\left(9.8\text{ m/s}^2\right)}} = 1.6\text{ s or } T \sim \boxed{10^0\text{ s}}.$$

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The angular position of a pendulum is represented by the equation . $\theta = (0.320\, rad) \cos \omega t$, where θ is in radians and $\omega = 4.43\, rad/s$. Determine the period and length of the pendulum.

$$\omega = \frac{2\pi}{T}: \qquad T = \frac{2\pi}{\omega} = \frac{2\pi}{4.43} = \boxed{1.42\text{ s}}$$

$$\omega = \sqrt{\frac{g}{L}}: \qquad L = \frac{g}{\omega^2} = \frac{9.80}{(4.43)^2} = \boxed{0.499\text{ m}}$$

A simple pendulum has a mass of 0.250 kg and a length of 1.00 m. It is displaced through an angle of 15.0° and then released. What are (a) the maximum speed, (b) the maximum angular acceleration, and (c) the maximum restoring force? **What If?** Solve this problem by using the simple harmonic motion model for the motion of the pendulum, and then solve the problem more precisely by using more general principles.

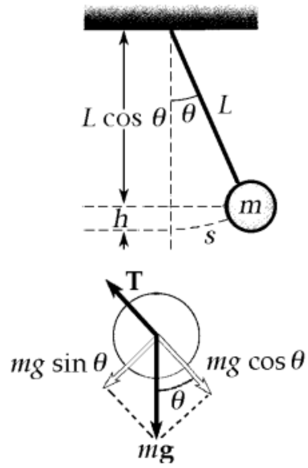


FIG. P15.31

Using the simple harmonic motion model:

$$A = r\theta = 1\ \text{m}\ 15° \frac{\pi}{180°} = 0.262\ \text{m}$$

$$\omega = \sqrt{\frac{g}{L}} = \sqrt{\frac{9.8\ \text{m/s}^2}{1\ \text{m}}} = 3.13\ \text{rad/s}$$

(a) $v_{\text{max}} = A\omega = 0.262\ \text{m}\ 3.13/\text{s} = \boxed{0.820\ \text{m/s}}$

(b) $a_{\text{max}} = A\omega^2 = 0.262\ \text{m}(3.13/\text{s})^2 = 2.57\ \text{m/s}^2$

$a_{\tan} = r\alpha \qquad \alpha = \frac{a_{\tan}}{r} = \frac{2.57\ \text{m/s}^2}{1\ \text{m}} = \boxed{2.57\ \text{rad/s}^2}$

(c) $F = ma = 0.25\ \text{kg}\ 2.57\ \text{m/s}^2 = \boxed{0.641\ \text{N}}$

More precisely,

(a) $mgh = \frac{1}{2}mv^2 \qquad$ and $\qquad h = L(1-\cos\theta)$

$\therefore v_{\text{max}} = \sqrt{2gL(1-\cos\theta)} = \boxed{0.817\ \text{m/s}}$

(b) $I\alpha = mgL\sin\theta$

$\alpha_{\text{max}} = \frac{mgL\sin\theta}{mL^2} = \frac{g}{L}\sin\theta_i = \boxed{2.54\ \text{rad/s}^2}$

(c) $F_{\text{max}} = mg\sin\theta_i = 0.250(9.80)(\sin 15.0°) = \boxed{0.634\ \text{N}}$

A very light rigid rod with a length of $0.500\ m$ extends straight out from one end of a meter stick. The stick is suspended from a pivot at the far end of the rod and is set into oscillation.
(a) Determine the period of oscillation.
Suggestion: Use the parallel-axis theorem.
(b) By what percentage does the period differ from the period of a simple pendulum $1.00\ m$ long?

(a) The parallel-axis theorem:

$$I = I_{\text{CM}} + Md^2 = \frac{1}{12}ML^2 + Md^2 = \frac{1}{12}M(1.00\ \text{m})^2 + M(1.00\ \text{m})^2$$

$$= M\left(\frac{13}{12}\ \text{m}^2\right)$$

$$T = 2\pi\sqrt{\frac{I}{Mgd}} = 2\pi\sqrt{\frac{M\left(13\ \text{m}^2\right)}{12Mg(1.00\ \text{m})}} = 2\pi\sqrt{\frac{13\ \text{m}}{12\left(9.80\ \text{m/s}^2\right)}} = \boxed{2.09\ \text{s}}$$

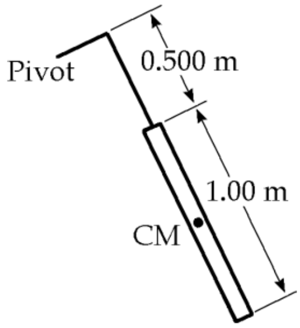


FIG. P15.36

(b) For the simple pendulum

$$T = 2\pi\sqrt{\frac{1.00\ \text{m}}{9.80\ \text{m/s}^2}} = 2.01\ \text{s}$$

$$\text{difference} = \frac{2.09\ \text{s} - 2.01\ \text{s}}{2.01\ \text{s}} = \boxed{4.08\%}$$

A torsional pendulum is formed by taking a meter stick of mass 2.00 kg, and attaching to its center a wire. With its upper end clamped, the vertical wire supports the stick as the stick turns in a horizontal plane. If the resulting period is 3.00 minutes, what is the torsion constant for the wire?

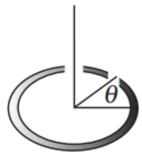
We suppose the stick moves in a horizontal plane. Then,

$$I = \frac{1}{12}mL^2 = \frac{1}{12}(2.00\text{ kg})(1.00\text{ m})^2 = 0.167\text{ kg}\cdot\text{m}^2$$

$$T = 2\pi\sqrt{\frac{I}{\kappa}}$$

$$\kappa = \frac{4\pi^2 I}{T^2} = \frac{4\pi^2\left(0.167\text{ kg}\cdot\text{m}^2\right)}{(180\text{ s})^2} = \boxed{203\ \mu\text{N}\cdot\text{m}}$$

A clock balance wheel has a period of oscillation of 0.250 s. The wheel is constructed so that its mass of 20.0 g is concentrated around a rim of radius 0.500 cm. What are (a) the wheel's moment of inertia and (b) the torsion constant of the attached spring?



$$T = 0.250 \text{ s}, \; I = mr^2 = \left(20.0 \times 10^{-3} \text{ kg}\right)\left(5.00 \times 10^{-3} \text{ m}\right)^2$$

(a) $I = \boxed{5.00 \times 10^{-7} \text{ kg} \cdot \text{m}^2}$

(b) $I \dfrac{d^2\theta}{dt^2} = -\kappa\theta; \; \sqrt{\dfrac{\kappa}{I}} = \omega = \dfrac{2\pi}{T}$

$$\kappa = I\omega^2 = \left(5.00 \times 10^{-7}\right)\left(\frac{2\pi}{0.250}\right)^2 = \boxed{3.16 \times 10^{-4} \; \frac{\text{N} \cdot \text{m}}{\text{rad}}}$$