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Faculty of **Mathematics** and **Computer Science**

Department of Computer science

Numerical methods exam

(1h30)

Exercice 1 (6 points)

Solve the following linear system using the simple Gaussian elimination method.

$$\begin{array}{rcrcrcl} & & x_2 & + & x_3 & = & 1 \\ x_1 & + & 2x_2 & + & 2x_3 & = & 1 \\ 2x_1 & + & x_2 & + & 2x_3 & = & 3 \end{array}$$

Exercice 2 (6 points)

Soit la matrice B :

$$B := \begin{pmatrix} 1 & 2 & 4 \\ -2 & 1 & 3 \\ -4 & -3 & 1 \end{pmatrix}$$

1- *Find an LU factorization of the matrix B, where L is the unit diagonal low triangular matrix and U is the upper triangular matrix.*

2- *Deduce the determinant of B*.

3- *Solve the equation* $BX = \begin{pmatrix} 5 \\ 7 \\ 9 \end{pmatrix}$. *Using LU factorization.*

Exercice 3 (8 points)

Consider the following linear system of equations

$$\begin{array}{rcrcrcr} 4x_1 & - & x_2 & + & x_3 & = & 12 \\ -x_1 & + & 3x_2 & + & x_3 & = & 1 \\ x_1 & + & x_2 & + & 5x_3 & = & -14 \end{array}$$

1- Show that both iterative methods (Jacobi and Gauss-Seidel) will converge by using *: (T=J* $\|T\|_\infty < 1$ *and T=G for Jacobi and Gauss-seidel matrix respectively)*

2- Find second approximation $\mathbf{x}^{(2)}$ when the initial solution is $\mathbf{x}^{(0)}$ = [4*;* 3*;* -3]*T for the both method (Jacobi and Gauss seidel)*

3- How many iterations (for the both method) needed to get an accuracy within 10^{-4}*:* using the formula

$$\|\mathbf{x} - \mathbf{x}^{(k)}\| \le \frac{\|T_J\|^k}{1 - \|T_J\|} \|\mathbf{x}^{(1)} - \mathbf{x}^{(0)}\| \le$$ ε (*(T=J and T=G for Jacobi and Gauss-seidel matrix respectively)*