USTO –MB 2023-2024 **MN Retake Exam**

Dept. Computer Science

LMD L2 (S3)

01h15

Section:…….Groupe……

First name:……Last name:……

Exercise 1 :

1- Solve the following linear system using the simple Gaussian elimination method:

2 x+ y- z = 1

X – 3 y + 2 z = 5

4 x + 2 y – 3 z = 3

2- Find an LU factorization of the matrix B, where L is the unit diagonal low triangular matrix and U is the upper triangular matrix

3- Deduce the determinant of A

1)- (4.5pts)=

Triangular (……/2.5 pts)

Matrix with B: 0.5pts

				L2= 0.5pts
				L3= 0.5pts

Matrix result 1 pts

				L3= 0.5pts

Resolution (……/2 pts)

Sys equation 0.5 pts : …… = ……..

…….. = ……

…… = ……

Solution 1.5 pt :

X=(…… , ……., ……)^t

2)- (5 pts)=

(……/ 2 pts)

L=			

(……/ 3 pts)

U=			

3)- (0.5 pts)=

Det A=……

Exercise 2 :

Consider the following linear system of equations

$$A = \begin{bmatrix} 4 & -1 & 0 \\ -1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

1-Show that both iterative methods, Jacobi and Gauss-Seidel, will converge by using the necessary and sufficient convergence conditions

2- How many iterations (for jacobi method) needed to get an accuracy within 10^{-2}:*with B=[2,5,1]*

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1)-(9 pts)=

(0.75 pts/……)

D=			

E=			

F=			

Jacobi (……/2pts)

Law J=…… ……..0.5pts

…….=			

……..=			

J=			

1 pts

Gaus siedal (……../2pts)

Law GS=……..0.5pts

……=			

……=			

GS=			

1 pts

Convergence Law ……(……/0.25pts)

Jacobi Case ……/2 pts :

……..=

……

……

Response……

Gauss-seidal ……/2 pts :

……..=

……

……

Response……

3)- (1pts)=

Response ……

Solution Exo 1 :(10 pts)

$$2x + y - z = 1$$

$$X - 3y + 2z = 5$$

$$4x + 2y - 3z = 3$$

1- [4,5 pts]

Triangularisation (2.5 pts)

(1.5 pts)

mat augmenté par B: 0.5pts

2	1	-1	1	
1	-3	2	5	L2=L2- 0.5 L1 0.5pts
4	2	-3	3	L3=L3-2 L1 0.5pts

(1 pts)

mat 1 pts (0.5 pts L2 et 0.5 pts L3)

2	1	-1	1	
0	-7/2	5/2	9/2	
0	0	-1	1	L3=L3

Résolution (2 pts)

Sys équation 0.5 pts :

$$2x + y - z = 1$$

$$-7/2\, y + 5/2\, z = 9/2$$

$$-z = 1$$

Solution 1.5 pt : $X=(1, -2, -1)$

2- (5 pts)

L (2 pts= 0.5 pts diagonal à 1, 0.5 pts pour chaque valeurs) et U(3 pts= 0.5 pts pour chaque valeurs)

3- (0.5 pts= 0.25 loi+0.25 valeur)

Det A= Det L*det U= 7

A=[2,1,-1;

1,-3,2;

4,2,-3]

L=[1 , 0, 0;

0.5, 1, 0;

2 , 0 , 1]

U=[2 , 1 , -1;

0, -3.5, 2.5;

0, 0 , -1]

Solution Exo 2:(10 pts)

$$A = \begin{bmatrix} 4 & -1 & 0 \\ -1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

(0.75 pts)= 0.25 pts chaque matrice

D=	4		
		2	
			3

E=			
	1		
	0	0	

F=		1	0
			0

Jacobi (2pts)

‘0.25 pts

InvD=	1/4		
		½	
			1/3

0.25 pts

E+F=	0	-1	0
	-1	0	0
	0	0	0

$J=D^{-1}(E+F)$……..0.5pts

J=	0	-1/4	0
	-1/2	0	0
	0	0	0

……1pts

Gaus siedal (2pts)

$GS=(D-E)^{-1}F$……0.5pts

D-E	4	0	0
	-1	2	0
	0	0	3

……..0.25pt

	¼	0	0

Inv D-E	1/8	1/2	0
	0	0	1/3

……..0.5pts

GS=	0	1/4	0
	0	1/8	0
	0	0	0

……1 pts

La loi de Convergence det(B-limda I)=0…….0.25 pts

Jacobi

J-Lamda I=	-lamda	-1/4	0
	-1/2	-lamda	0
	0	0	-lamda

……0.25 pts

Det =lam^3+0.25(-0.5 lam)=Lam(lam^2-1/8)=0……..0..25

Lam=0 Lam=racine(1/8) Lam=-racine(1/8) 0.75

Rayon =max (abs lam)= racine(1/8) <1 converge……..0.5 pts+0.25 pts

Gauss Siedal

GS-Lamda I=	-lamda	1/4	0
	0	1/8-Lamda	0
	0	0	-Lamda

……0.25 pts

Det = lam^2 (1/8- lam)=Lam(lam^2-1/8)=0……..0..25

Lam=0 Lam=racine(1/8) Lam=-racine(1/8) 0.75

Rayon =max (abs lam)= racine(1/8) <1 converge……..0.5 pts+0.25 pts

La 3 question (1pts)